Adding Forces (Ch 6)

Components of a Force

\[ F = 316 \, @ \, 35^\circ \]

\[ F_x = 316 \times \cos(35) \]

\[ \cos(35) = 0.81915 \]

\[ F_x = 316 \times 0.81915 = 258.8514 \text{ N} \]

\[ F_y = 316 \times \sin(35) = 181.2502 \text{ N} \]

In Autocad we can double-click the line to get x & y components.

Order does not matter, but direction of forces must be correct.

Here we use Autocad to add forces as lines. Resultant must start from same start.
Forces as lines. Resultant must start from same start point.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta X</td>
<td>-489.4171</td>
</tr>
<tr>
<td>Delta Y</td>
<td>2544.7516</td>
</tr>
<tr>
<td>Delta Z</td>
<td>0</td>
</tr>
<tr>
<td>Length</td>
<td>2501.3877</td>
</tr>
<tr>
<td>Angle</td>
<td>100.886</td>
</tr>
</tbody>
</table>
**Example: A Lifting Eye**
Two ropes are attached to this lifting eye. Force A is 1200N at 75°, and Force B is 1600N at 60° from horizontal.
What is the RESULTANT of these two forces?
Q14: Find X component (Fx) if FA=58kN and angle B=29 degrees.

\[
Fx = 58 \cdot \cos(29) = 50.728 \text{ kN}
\]
\[
Fy = 58 \cdot \sin(29) = 28.119 \text{ kN}
\]

Q16: Find the MAGNITUDE of the Resultant force if Rx=373N and Ry=518N.

Pythagoras:

\[
Res = \sqrt{Fx^2 + Fy^2}
\]
\[
= (373^2 + 518^2)^{0.5} = 638.3205 \text{ N}
\]

Q17: Find the ANGLE of the Resultant force if Rx=348N and Ry=670N.

\[
\text{Angle} = \arctan \left( \frac{670}{348} \right) = 62.552 \text{ degrees}
\]
Q4: Find Rx (Resultant in X direction) where forces A=13kN, B=24kN, C=22kN and D=14kN.

Put in 360 degree notation...
Fa = 37+73 = 110°
Fb = 73°
Fc = 360-17 = 343°
Fd = 360-17-118 = 225°

Get components in X direction
Fx = F * cos (angle)
Fax = 13 *cos(110) = -4.4463 kN
Fbx = 24 *cos(73) = 7.0169 kN
Fcx = 22 *cos(343) = 21.039 kN
Fdx = 14 * cos(225) = -9.8995 kN
Sum of X components
ΣFx = -4.4463+7.0169+21.039-9.8995 = 13.7101 kN

Get components in Y direction
Fy = F * sin (angle)
Fay = 13 *sin(110) = 12.216 kN
Fby = 24 *sin(73) = 22.951 kN
Fcy = 22 *sin(343) = -6.4322 kN
Fdy = 14 * sin(225) = -9.8995 kN
Sum of Y components
ΣFy = 12.216+22.951-6.4322-9.8995 = 18.8353 kN

Resultant?
Pythagoras:
Res = \sqrt{Fx^2 + Fy^2}

= (13.7101^2 + 18.8353^2) ^ 0.5 = 23.2967 kN
Angle = \( \arctan \left( \frac{18.8353}{13.7101} \right) = 53.949^\circ \)

What happens if it was in the third quadrant?

Angle = \( \arctan \left( \frac{-18.8353}{-13.7101} \right) = 53.949^\circ \)

Doesn’t work! Atan gets mixed up between quadrants 1 & 3 and also quadrants 2 & 4. So DRAW the force triangle.
Adding Forces

360 degree notation

180° 47N @ -155°

Works too

Positive trig functions

±180° notation

Alternative also works

47N @ -155°

⊕ No mistakes
⊕ No diagram
⊕ Automatic negative signs in EXCEL/COMPUTER PROGRAMS/CAD

65° (1st quadrant)

A = AM
\[ A = \text{AM} \]
\[ s = \sin (65°) \]
\[ t = \tan (65°) \]
\[ c = \cos (65°) \]

135° (2nd quadrant)
\[ F_x = F \cos \theta \quad F_y = F \sin \theta \]

\( \text{Eg.} 47 \text{N@205°} \)

\[ F_x = 47 \times \cos(205°) = 47 \times -0.9063 = -42.6 \text{ N} \]

\[ F_y = 47 \times \sin(205°) = 47 \times -0.4226 = -19.86 \text{ N} \]
Adding Forces 2

Monday, 13 February 2012  8:06 PM

\[ F_2 = 4 \]

\[ F_1 = 3N @ 90^\circ \]

\[ F_2 = 4N @ 0^\circ \]

\[ \tan \theta = \frac{3}{4} \]

\[ \tan^{-1} \left( \frac{3}{4} \right) = 36.87^\circ \]

Resultant: \[ R = 5N @ 36.87^\circ \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 - 2 \mathbf{b} \mathbf{c} \cos \theta \]

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
1. Split forces into components
2. Add components to get total $\Sigma F_x$ and total $\Sigma F_y$
3. Add $\Sigma F_x$ and $\Sigma F_y$ to get resultant

This works for any number of forces

Magnitude of Resultant:

$$R = (\Sigma F_x^2 + \Sigma F_y^2)^{0.5}$$

$$R = (-489.417^2 + 2544.75^2)^{0.5} = 2591.386$$

Angle of Resultant:

$$\text{Angle} = \tan^{-1} (\Sigma F_y/\Sigma F_x)$$

$$\text{Angle} = \tan^{-1}(2545.2/-489.42) = -79.1154$$

Correct answer: $180 - 79.1154 = 100.8846$

Pythagoras: $a^2 + b^2 = c^2$
Correct answer only from:
EXCEL: \text{atan2}(x,y)
not
EXCEL: \text{atan}(y/x)

Circum = 2\pi R
360^\circ = 2\pi R \text{ length}
1 \text{ radian} = \frac{360}{2\pi} = \frac{180}{\pi}
180/\text{Pi} = 57.29578^\circ
Adding Forces by Components 2

1. Put into 360 notation!

2. Now get components...
   \[ F_x = \text{Magnitude} \times \cos (\text{Angle}) \]
   \[ F_y = \text{Magnitude} \times \sin (\text{Angle}) \]

3. Now add these together to get \( R_x \) and \( R_y \)
   \[ R_x = F_{1x} + F_{2x} + F_{3x} + \ldots \]
   \[ R_y = F_{1y} + F_{2y} + F_{3y} + \ldots \]

4. Get the Resultant magnitude
   \[ R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \]

5. Get the Resultant angle
   \[ \theta = \arctan\left(\frac{\Sigma F_y}{\Sigma F_x}\right) \]

**Example 4.9c**

1. \( F_1 = 10\, \text{kN@225}^\circ \)
   \( F_2 = 6\, \text{kN@120}^\circ \)
   \( F_3 = 16\, \text{kN@210}^\circ \)
   \( F_4 = 20\, \text{kN@330}^\circ \)

2. \( F_{1x} = 10\cos(225) = -7.071\, \text{kN} \)
   \( F_{2x} = 6\cos(120) = -3.00\, \text{kN} \)
   \( F_{3x} = 16\cos(210) = -13.856\, \text{kN} \)
   \( F_{4x} = 20\cos(330) = 17.321\, \text{kN} \)
   \[ \text{TOTAL} \, F_x = -7.071 - 3.00 - 13.856 + 17.321 = -6.606 \, \text{kN} \]
   \( F_{1y} = 10\sin(225) = -7.071\, \text{kN} \)
   \( F_{2y} = 6\sin(120) = 5.196\, \text{kN} \)
   \( F_{3y} = 16\sin(210) = -8.000\, \text{kN} \)
   \( F_{4y} = 20\sin(330) = -10.000\, \text{kN} \)
   \[ \text{TOTAL} \, F_y = -7.071 + 5.196 - 8 - 10 = -19.875 \, \text{kN} \]

\[ R = (\sqrt{(-6.606)^2 + (-19.875)^2})^{0.5} = 20.9441 \, \text{kN} \]