Alternatively:
\[ \eta = \frac{MA}{VR} \]
\[ = \frac{9}{12} \]
\[ = 0.75 \]
\[ = 75\% \]

### 22.3 Friction Effort

If the machine were perfect, no work would have to be done against friction, and the efficiency would be 100 per cent. This would mean that for a perfect machine:

\[ \frac{MA}{VR} = 100\% \]

or that the ideal mechanical advantage is equal to the velocity ratio:

\[ MA = VR \]

It also means that if there were no friction to be overcome, it would take a smaller effort to move the same load. The effort required to move a given load \( F_L \) if the machine is 100 per cent efficient is called the theoretical effort, \( F_{th} \). Substituting into \( MA = VR \), we have:

\[ \frac{F_L}{F_{th}} = VR \quad \text{or} \quad F_{th} = \frac{F_L}{VR} \]

The difference between the actual effort \( F_E \) and the theoretical effort \( F_{th} \) is the effort wasted in overcoming friction and is known as the frictional effort, \( F_F \).

\[ F_F = F_E - F_{th} \]

#### Example 22.3

For the machine in the previous examples, calculate the theoretical and frictional efforts.

**Solution**

Theoretical effort:

\[ F_{th} = \frac{F_L}{VR} \]
\[ = \frac{450}{12} \]
\[ = 37.5 \text{ N} \]

Frictional effort:

\[ F_F = F_E - F_{th} \]
\[ = 50 \text{ N} - 37.5 \text{ N} \]
\[ = 12.5 \text{ N} \]

### 22.4 THE LAW OF A MACHINE

The law of a machine is an equation which expresses the relationship between load \( F_L \) and effort \( F_E \). In many cases this relationship, when plotted as a graph of effort against load, is a straight line. Its mathematical equation is of the linear form:

\[ F_E = aF_L + b \]

where \( F_E \) is the effort,
\( F_L \) is the load,
\( a \) is the slope of the graph,
\( b \) is the value of \( F_E \) where the graph cuts the \( F_E \) axis.

After the constants have been determined for a particular machine, the law of the machine can be used to predict the effort required to move any load by the machine.

**Example 22.4**

The machine in the previous examples was tested under different loads, and the following efforts were recorded for each of the loading conditions:

<table>
<thead>
<tr>
<th>Load ( F_L ) (N)</th>
<th>0</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effort ( F_E ) (N)</td>
<td>5</td>
<td>25</td>
<td>45</td>
<td>65</td>
<td>85</td>
<td>105</td>
</tr>
</tbody>
</table>

Plot the load–effort graph and determine the law of the machine. Use the law to estimate the effort required to move a load of 700 N.

**Solution**

![Fig. 22.2](image)

In Figure 22.2, which is the load–effort graph, the line cuts the effort axis at \( F_E = 5 \). This is the value of \( b \).